

Naturally Mathematical 2010

Junior Solutions 1

Question 1: Fill the Blanks

Part 1

Fill the blanks with the odd digits 1, 3, 5, 7 and 9 to make the multiplication correct.

Part 2

Fill the blanks with the consecutive digits 1, 2, 3, 4 and 5 to make the multiplication correct.

Part 3

Fill the blanks with the even digits 2, 4, 6, and 8 and one odd digit to make the multiplication correct.

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Solution 1: Einsteins (Nambour SS)

QUESTION 1A

Sort out

- * We had to use the digits 1,3,5,7 and 9 in a multiplication sum using all the digits once.
- * We had to multiply a two digit number by a one digit number to get a two digit number

Think about

We know that if you multiply a 2 digit number by 1 the answer will be the same as the 2 digit number and we can only use the numbers once.

When we multiply by 5 the answer will end in 5 or 0 – we haven't got a 0.

If we multiply the smallest number by 9 we think the answer would probably be bigger than 100 which is too big.

Action

We used 13 the smallest number and multiplied it by the other numbers. It didn't work.

So we tried 19×3 and it equalled 57 – all the numbers fitted in.

Reflect on ...

I think it was quite hard but we feel pleased that I finished it.

QUESTION 1B

Sort Out:

We had to do the same thing with different numbers – 1,2,3,4 and 5.

Think about:

First we multiplied the smallest 2 digit number by the highest 1 digit number possible. We knew from previous experiences that any number multiplied by 5 would end in 5 or 0.

Action:

We multiplied the smallest 2 digit number which was 12 by 4. That was wrong because we couldn't use 4 twice and we didn't have an 8. So we added 1 to make 13 and multiplied by 4 the answer was 52 and we had used all the digits!

Reflect on:

Our strategy worked really well. We think the question wasn't terribly hard but not really easy either.

QUESTION 1C

Sort out

We had to do a 2 digit multiplication, multiplied by a 1 digit number with the numbers 2,4,6,8 and any odd number.

Think about

We knew how to do 2 digit by 1 digit multiplication.
We figured out the 1 wasn't useful for anything
The 7 wasn't useful for anything and the 9 wasn't either.

Action

We used the low numbers first – they didn't work.
Then we used a high & low digit number, 4 & 8, and then multiplied by 2.
Our estimation was 4 more than our answer. We tried multiple combinations with high numbers – they didn't work so we tried low numbers – that worked! 34 multiplied by 2 was 68.

Reflect on:

Our strategy worked out after a while.

Note:

I had found this answer when preparing the question, but other teams found two more, $43 \times 2 = 86$ and $48 \times 2 = 96$. It was great to be shown the way!

Question 2: Paths from 1 to 9

You have to find a way to go from the square labelled 1 to the square labelled 9. You can move in any direction (horizontally, vertically or diagonally) so long as you move into a square where the number is larger than the one you are currently in.

How many different paths are there from 1 to 9?

| | | |
|---|---|---|
| 1 | 3 | 6 |
| 2 | 5 | 8 |
| 4 | 7 | 9 |

Solution 2: Maths Master (Indooroopilly SS)

Sort out: The important information was how many different ways we could get from 1 to 9 without descending in numbers. The other important part of the information was that you could move vertically, horizontally and diagonally. We knew there was no missing information as we had checked many times. We had a dictionary handy in case we came across any difficult words that we didn't know.

Think about: Our strategy was to do all the ways you can do one number at a time. We have never done a problem the same or similar to this problem before so we found this one the most difficult. We used scrap paper to write down all the answers and used another to write it down neater so we could read it.

Action: We checked that we used all the given information and are positive we didn't miss out any information. Once we had quite a few answers we started checking that we hadn't done it before. Once we had finished, we checked to see if we didn't do it twice.

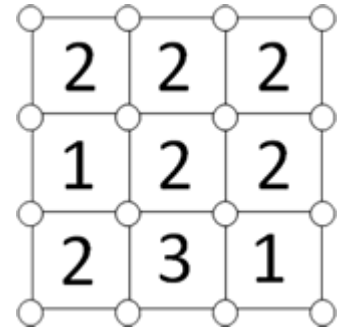
| | | | | | |
|----|---------|----|--------|----|---------|
| 1 | 13689 | 12 | 124789 | 23 | 15789 |
| 2 | 13589 | 13 | 12479 | 24 | 1579 |
| 3 | 135689 | 14 | 125689 | 25 | 15689 |
| 4 | 135789 | 15 | 12579 | 26 | 12389 |
| 5 | 13579 | 16 | 125789 | 27 | 123689 |
| 6 | 1359 | 17 | 12589 | 28 | 1235689 |
| 7 | 1389 | 18 | 1259 | 29 | 123589 |
| 8 | 1245789 | 19 | 12789 | 30 | 1235789 |
| 9 | 12479 | 20 | 1279 | 31 | 123579 |
| 10 | 124589 | 21 | 1589 | 32 | 12359 |
| 11 | 12459 | 22 | 159 | 33 | 1245689 |

Reflect on: We thought our strategy was a good one because it worked well for us.

If we get another problem similar to this we would use the same strategy. We are happy with our answer and very confident about it.

Question 3: Corner Circles

In this 3×3 grid, the circles at the corner of each square have to be coloured either black or white. The number in the centre of each square tells how many of the circles at its corners should be coloured black.



Be a STAR problem solver

Click on the diagram above to start an applet that gives you lots of examples of this problem. You can use this applet to help you **Sort out** what the question is all about and to **Think about** a strategy for finding which circles need to be coloured black.

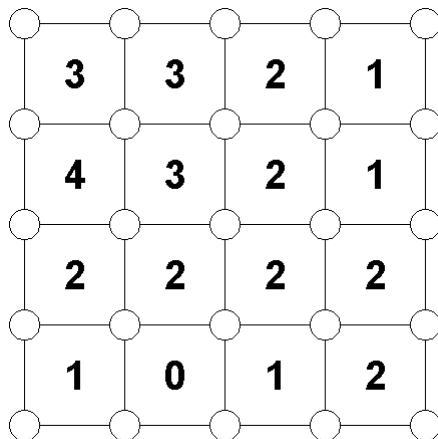
Action and Reflection

Click on the link below to start the applet for a 4×4 grid and use two examples of the problem that you solved to explain any strategies that you found helped you solve the problem.

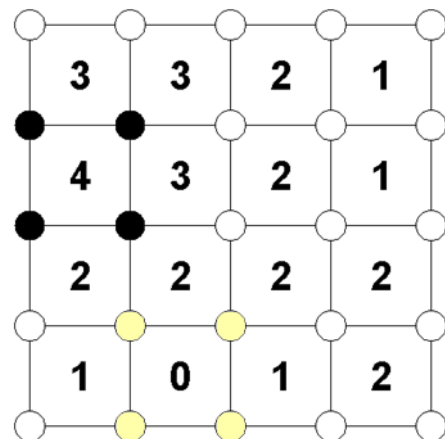
Solution 3

This problem was new to all the teams – “We have never done a problem like this before!” But that didn’t stop them from clearly understanding what the applet did and most appeared to become very proficient at solving the puzzles that the applet generated.

The following puzzle has the features that almost everyone commented on:

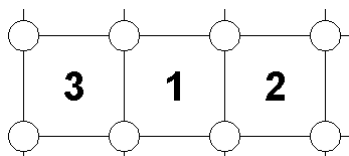


Where there is a 4, fill in all the corners of the square and where there is a zero, make sure to leave the corners empty.

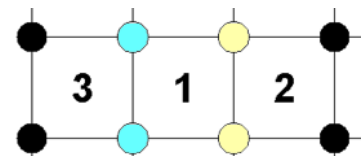


Indeed, quite a lot of this particular puzzle can be completed by making logical steps following on from this initial position.

Another pattern that fixes the position of the black circles is $3 - 1 - 2$:



The end four circles can immediately be turned black and there has to be one black circle between the 3-square and 1-square.



If you are wondering “Where’s the Maths?”, don’t worry. This puzzle calls for adaptive reasoning, the skill of saying “if this, then that” which is crucial to the development of mathematical thinking. And of course there is the important confidence-building aspect as the students are constantly being asked to venture into new territories and they need the confidence to have a go, even if it is in the ‘never done a problem like this before’ category.