



naturally
mathematical

Building Multiplicative Strategies (BMS)

**A Collaborative Guided Inquiry
with Ann Baker**

**Series
Introduction**





Overview

Multiplicative thinking has many facets which include strategies, representations and contexts. This package covers this topic comprehensively, even though many of the components can be used separately when the teacher sees a need for a particular aspect that needs attention. The package consists of:

1. Strategy lessons:
 - Strategy 1: Flexible Thinking
 - Strategy 2: Double
 - Strategy 3: Number Splitting
 - Strategy 4: Using 10s Facts
 - Strategy 5: Landmark Numbers
 - Strategy 6: Factors
2. Mental routines
3. Problematized situations
4. Word problems
5. Number talk
6. Calculator challenges

The material is also set within the context of an inquiry that will enable the teacher to show where the students started from and where they have reached in their exploration of multiplicative thinking.





Having an Impact?

John Hattie has coined the term 'know thy impact'. Teachers are more and more being expected to know if they are having sufficient impact on their students' learning. Teachers are also expected to involve students in knowing the learning intentions and to be involved in knowing how well they are meeting the success criteria. With this in mind we have planned this mini-classroom based inquiry such that we can begin to collect data about our impact, what's working, what we can achieve and how we can share the learning journey with our students.

The Collaborative Guided Inquiry

An inquiry begins with important guiding questions. I have outlined two questions below but as you work with peers you may want to rewrite them to suit your own needs or interests. At least they will start the process.

Acknowledgement

Johnny and I would like to express our very sincere thanks to Caroline Swinburne for her support in the preparation of this series. “Caroline, thanks to your insights and comments, parts of the series that might have been incomprehensible are now accessible to all!”



Questions to ask yourself

Question 1

To what extent do I and my students use fluent and flexible strategies for mental computation of multiplication?

Question 2

Will immersion in thinking *smart* not *hard* enhance actual multiplication strategy choice and disposition towards working multiplicatively?

What do we know about students and multiplicative thinking?

We know that some of our students:

- exit primary school still thinking additively
- have a fixed mindset that limits their willingness to move away from known procedures when other strategies would save brain space
- find satisfaction and enjoyment in exploring efficient strategies even if they can carry out a rote procedure.



The Learning Intentions

Intention 1

To use dialogue and visual connections to encourage students to develop a deep understanding of working *smart* not *hard* when doing a multiplicative calculation.

Intention 2

To develop pedagogy that promotes engagement with, and disposition toward, fluency and flexibility as outlined in the ACARA: National Numeracy Learning Progressions.

Activities for the Inquiry

Observations, work samples and anecdotes will be collected lesson by lesson so that we can examine our impact through ongoing formative assessment.



Getting Started: A quick check

Before moving on to the actual focus and pedagogical content goals of this inquiry please look at the following multiplication question and notice how you go about answering it.

What is 4×13 ?

We are going to ask you to try this with your class, but before you do, look at and consider what approaches and starting points you might find students in your class using. You might also like to consider which category you fit in:

- **working hard**
- **working procedurally**
- **working smart.**

Examples of these are on the next two pages.

Chances are if you were over-taught rote procedures, working procedurally will be the one you like most. Students with fixed mindsets often refuse to try alternative methods such as in Samples 6, 7 and 8. They will say "I know how to do this. I always do it this way."

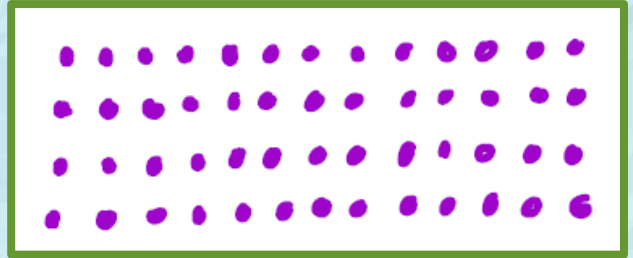


Working Hard

Students who are working hard may well present any of the following approaches, each of which is additive.

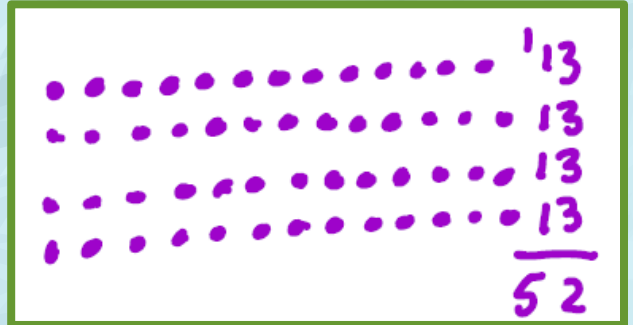
Sample 1

Draw and count all.



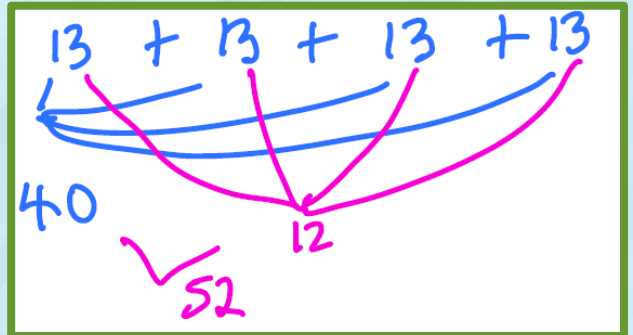
Sample 2

Draw and count each row (not trusting the count) and then using a traditional method



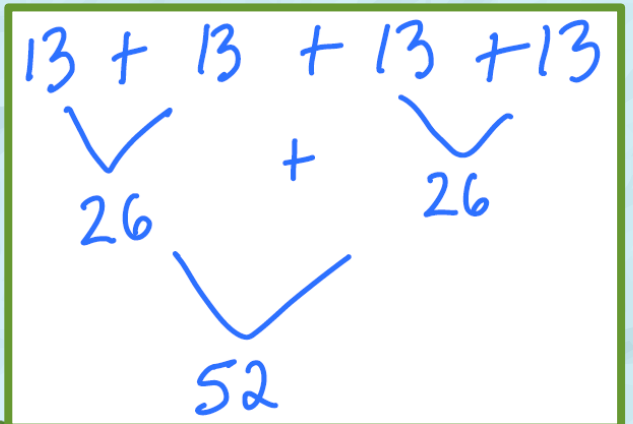
Sample 3

Using number splitting, seeing 13 as 10 and 3 more, then adding 10s then 3s and then $40 + 12$



Sample 4

Shows a transition between additive and multiplicative thinking and uses a *double-double*.



Working Procedurally

Sample 5

Using known facts and a familiar procedure.

$$\begin{array}{r} 13 \\ \times 4 \\ \hline 52 \end{array}$$

Working Smart

Sample 6

Using the distributive property, 13 is $10 + 3$ fluently and flexibly

$$\begin{array}{l} 4 \times 10 = 40 \\ 4 \times 3 = 12 \end{array} \left. \vphantom{\begin{array}{l} 4 \times 10 = 40 \\ 4 \times 3 = 12 \end{array}} \right\} 52$$

Sample 7

Using factors, 4 is 2×2 , similar to the double-double in Sample 4 but applied multiplicatively and abstractly

$$\begin{array}{l} 2 \times 13 = 26 \\ 2 \times 13 = 26 \end{array} \left. \vphantom{\begin{array}{l} 2 \times 13 = 26 \\ 2 \times 13 = 26 \end{array}} \right\} 52$$

Sample 8

Making connections, 15 minutes in quarter of an hour, $4 \times 15 = 60$ minutes, so round 13 up to 15 then adjust.

$$\begin{array}{l} 4 \times 15 = 60 \\ 4 \times 2 = 8 \\ 60 - 8 = 52 \end{array}$$



ACARA: Numeracy Learning Progressions



Samples 1, 2, 3 and 4 correspond with:

MuS4 - Repeated abstract composite units

- uses composite units in repeated addition and subtraction using the unit a specified number of times

Where does all this fit with the NLPs?

Sample 5 does not match the progressions, which are focussed on deep understanding of multiplication leading to flexible and efficient strategies rather than procedures.



Samples 6, 7 and 8 correspond with:

MuS5 - Coordinating composite units

- coordinates two composite units (mentally) as an operation. That is, both the number of groups and the number in each group are treated as composite units.
- represents multiplication in many ways (arrays, factors, product of combinations)

MuS6 - Flexible strategies for multiplication

- draws on the structure of multiplication to use known multiples in calculating related multiples
- uses known single-digit multiplication
- applies known facts and strategies for multiplication to mentally calculate
- uses commutative properties of numbers

MuS7 – Flexible number properties

- uses factors of a number to carry out multiplication and division
- uses the distributive property of multiplication over addition as in $6 \times (5 + 4) = 6 \times 5 + 6 \times 4$.